Modeling Exponential Growth and Decay of Fish in a Pond Using Skittles “Lab”

Goals/Objectives:

* Students will be able to graph exponential growth/decay data.
* Students will be able to solve exponential and linear regressions on the calculator.
* Students will collect and record data in a chart.
* Students will understand the difference between exponential growth and decay.

Materials:

* Exponential Growth and Exponential Decay of Fish in a Pond Worksheet
* 30 Skittles (with a clearly printed “S”)
* 1 Plastic Cup
* 1 Tray
* Graphing Calculator
* Extra Skittles (for snacking!)

This worksheet is designed to be an introduction to exponential functions. The main purpose of this lab is to allow students to understand the concepts of exponential growth and decay, as well as its real-world applications. This lab also reinforces their data collecting and graphing skills and requires them to analyze the data they collect.

**Overview for Part 1:**

A fisherman puts two fish in one pond. There are no natural predators in this pond, so the fish keep reproducing without any disturbance. Use skittles to help determine the number of fish in the pond, then determine the equation that best fits your data. Each trial represents a year.

**Overview for Part 2:**

A pond has 30 fish, which is its maximum capacity. Unfortunately, a deadly bacterium (to fish) is introduced into the pond and the fish begin dying at a rapid rate. Use skittles to help determine how many years it will be until there are no more fish left in the pond.

**What is a linear equation and why do we use it to model trends?**

A linear equation is one with the general form of $y=ax+b$. We use it to model trends that are increasing at a constant rate. For example, if you are told that a population of fish is increasing according to the formula y=3x+2 every year, we can say that the starting population, b, was 2 and it is increasing by 3 fish per year.

**What is an exponential equation and why do we use it to model trends?**

An exponential equation is one with the general form of $y=a\*b^{x}$**.** We use it to model trends that are increasing at an exponential rate. For example, if you are told that a population of fish is increasing according to the formula $y=2\*3^{x}$ every year, we can say that the starting population, a, is 2 and it increases by 3 times every year, meaning the rate of increase is not constant. This is useful when modeling a population that is growing faster and faster every year.

Example of how to add data into your calculator and find the exponential regression of best fit:

|  |  |
| --- | --- |
| **Trial #** | **Number of Skittles** |
| 0 | 2 |
| 1 | 6 |
| 2 | 18 |
| 3 | 54 |

To enter this data into the calculator, follow these steps:

1. Press [STAT] then [ENTER]
2. Enter the trial number column into the L1 column, pressing enter after each entry.
3. Enter the number of skittles into the L2 column, making sure that the data points line up correctly (ex: 0 in L1 and 2 in L2)

This is what you will see when you follow these steps on your calculator. Enter the trial number in L1 and number of skittles in L2.



To find the exponential regression, follow these steps: \*Note: you will also be finding the linear regression, but for this example, we will just find the exponential regression\*

1. Press [STAT] then arrow over to [CALC]
2. Arrow down until you see [ExpReg] and click it
3. Arrow down to [CALCULATE] and press [ENTER]

This is what you will see when you type in your data, go to calc and find ExpReg, and click calculate to get the a and b values.


When we do this, we find that our a=2 and b=3, meaning that our regression is $y=2\*3^{x}$.

**Part 1: Modeling Exponential Growth of Fish in a Pond Worksheet**

Procedure:

1. Gather all of your materials. Count out 30 Skittles and place them on the desk in front of you.
2. Place two skittles in your plastic cup and record them as Trial #0 in your table. These are the “mom and dad” fish. \*Note: Every new trial is a new “year”\*
3. Roll the skittles into your tray. Put one additional skittle into the tray for every skittle with the “S” showing. Count up the number of skittles in the tray and record it in the next trial number in the chart.
4. Repeat step 3 until you have completed 10 trials or you have run out of skittles. If you do not have enough skittles for your last trial, write in the table the number of skittles you SHOULD have in your tray.

|  |  |
| --- | --- |
| Trial # (Year) | Number of Skittles (Fish in the Pond) |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |

Plot the data points from your table above, making sure to label your axis. Connect your points with a curve.

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Now, let’s solve for the equation of this curve! We will start by entering the data on our graphing calculator! To do this, follow these steps:

1. Press [STAT] then [ENTER]
2. Enter the trial number column into the L1 column, pressing enter after each entry.
3. Enter the number of skittles into the L2 column, making sure that the data points line up correctly (ex: 0 in L1 and 2 in L2)

Once all of your data is entered, we will find the linear and exponential regressions.

To find the linear regression, follow these steps:

1. Press [STAT] then arrow over to [CALC]
2. Press [4] or [LinReg(ax+b)]
3. Make sure Xlist:L1 and Ylist:L2 is shown, if it is not, enter either L1 or L2 then arrow down to calculate and press enter

Write down the a and b values that your calculator populated, then put those values into the equation y=ax+b:

a=

b=

y=

To find the exponential regression, follow these steps:

1. Press [STAT] then arrow over to [CALC]
2. Arrow down until you see [ExpReg] and click it
3. Arrow down to [CALCULATE] and press [ENTER]

Write down the a and b values that your calculator populated, then put those values into the equation $y=a\*b^{x}$:

a=

b=

y=

Questions:

1. Graph the two equations you just wrote on the same set of axes on your calculator. Which of these two graphs looks most like the graph that you drew for the fish? Why do you think this is the case?
2. How many years did it take for the population of fish to pass 30?
3. Will the population ever stop growing? Why or why not? (Use your math and reasoning skills to answer this question).

**Part 2: Modeling Exponential Decay of Fish in a Pond Worksheet**

Procedure:

1. Gather all of your materials. Count out 30 Skittles and place them on the desk in front of you.
2. Place all 30 skittles in your plastic cup and record them as Trial #0 in your table. \*Note: every trial is a new “year” \*
3. Roll the cup of skittles onto the tray. Remove all skittles with the “S” showing. (These are fish that were infected with the bacteria and died.) Record the new number of skittles (fish remaining) in your chart.
4. Repeat step 3 until you have completed 10 trials or you have run out of skittles in the tray.

|  |  |
| --- | --- |
| Trial # | Number of Skittles (Fish in the Pond) |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |

Plot the data points from your table above, making sure to label your axis. Connect your points with a curve.

****

Now, let’s solve for the equation of this curve! We will start by entering the data on our graphing calculator! To do this, follow these steps:

1. Press [STAT] then [ENTER]
2. Enter the trial number column into the L1 column, pressing enter after each entry.
3. Enter the number of skittles into the L2 column, making sure that the data points line up correctly (ex: 0 in L1 and 2 in L2)

Once all of your data is entered, we will find the linear and exponential regressions.

To find the linear regression, follow these steps:

1. Press [STAT] then arrow over to [CALC]
2. Press [4] or [LinReg(ax+b)]
3. Make sure Xlist:L1 and Ylist:L2 is shown, if it is not, enter either L1 or L2 then arrow down to calculate and press enter

Write down the a and b values that your calculator populated, then put those values into the equation y=ax+b:

a=

b=

y=

To find the exponential regression, follow these steps:

1. Press [STAT] then arrow over to [CALC]
2. Arrow down until you see [ExpReg] and click it
3. Arrow down to [CALCULATE] and press [ENTER]

Write down the a and b values that your calculator populated, then put those values into the equation $y=a\*b^{x}$:

a=

b=

y=

Questions:

1. Graph the two equations you just wrote on the same set of axes on your calculator. Which of these two graphs looks most like the graph that you drew for the fish? Why do you think this is the case?
2. How many years did it take for all of the fish in the pond to die? Are you surprised by this?
3. What other factors might lead to the decay of fish besides this bacterium?

Wrap-Up Questions:

1. What do you notice about the similarities and differences of the a and b values of the exponential regressions for parts 1 and 2?
2. What do you notice about the similarities and differences of the a and b values of the linear regressions for parts 1 and 2?
3. What are three unique things about an exponential equation that cannot be observed in a linear equation?